

Exercise 2 – Z notation (2/CS/3T)

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1 Relations and Functions

1. Given the following Unix file permissions and sizes:

$$\begin{aligned} \textit{permission} &= \\ &\{ \textit{.login} \mapsto \text{-rw-r--r--}, \textit{.cshrc} \mapsto \text{-rw-r--r--}, \\ &\textit{Mail} \mapsto \text{drwx-----}, \textit{a.out} \mapsto \text{-rwxr-xr-x}, \textit{mbox} \mapsto \text{-rw-----} \} \\ \\ \textit{size} &= \\ &\{ \textit{.login} \mapsto 100, \textit{.cshrc} \mapsto 200, \textit{Mail} \mapsto 512, \textit{a.out} \mapsto 30000, \textit{mbox} \mapsto 40000 \} \end{aligned}$$

Calculate the following, and explain what they are in English:

- (a) $\text{dom}(\textit{size} \triangleright \{n : \mathbb{N} \mid n \leq 1024\})$
- (b) $\{\textit{a.out}\} \triangleleft \textit{size}$
- (c) $\textit{permission} \sim (| \{ \text{-rw-r--r--} \} |)$
- (d) $\textit{permission} \sim \S \textit{size}$
- (e) $\max(\text{ran } \textit{size})$
- (f) $\textit{permission} \oplus \{ \textit{.login} \mapsto \text{-rw-----}, \textit{.cshrc} \mapsto \text{-rw-----} \}$

2. Note that the infix overriding operator $_ \oplus _$ may be defined between two relations as follows: $R_1 \oplus R_2 = ((\text{dom } R_2) \triangleleft R_1) \cup R_2$

Explain the effect of the overriding operator in English.

Consider the use of overriding on functions (e.g., f_1 and f_2 below):

- (a) If $f_1 \oplus f_2 = f_1$, what can you say about f_2 ? Include your reasoning in your answer.
- (b) If $f_1 \oplus f_2 = f_2$, what can you say about f_1 ? Again, include your reasoning.
- (c) When does $f_1 \oplus f_2 = f_1 \cup f_2$ and why?

P.T.O.

2 Numbers and Sequences

- What is $\max\{n : \mathbb{N} \mid n < 1024\}$ and what is $\min\{n : \mathbb{N} \mid n < 1024\}$?
 - Rewrite $\{n : \mathbb{N} \mid n < 1024\}$ using “...”.
 - Formally define the factorial of a number $n!$ using an axiomatic definition, where (informally) $n! = n * (n - 1) * \dots * 2 * 1$. (Hint: use an inductive definition.)
- Simplify the following:
 - $\text{head rev}\langle Z \rangle$
 - $\text{tail front rev}\langle D, A, T, E \rangle$
 - $\text{rev}(\text{rev}(\langle M, A, D, A, M \rangle) \frown \langle I, M \rangle \frown \langle A, D, A, M \rangle)$
 - $\text{tail}(\frown / \langle \langle I \rangle, \langle O \rangle, \langle U \rangle \rangle) \frown \langle T \rangle$
- Define *last s* (the last element in a sequence) and *front s* (all elements in a sequence except the last one) in terms of *head*, *tail* and *rev*. In each case, what are the limitations on the sequence *s* for these definitions to be valid?

Note that information on the course is available on-line on the World Wide Web under the following URL (Uniform Resource Locator):

<http://www.cs.reading.ac.uk/cs/people/jpb/teaching/z.html>