

Exercise 3 – Z notation (2/CS/3T)

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Normalization and Schemas

1. Normalize the following expressions to the largest possible supersets, giving their types in terms of the power set operator ‘ \mathbb{P} ’, the Cartesian product ‘ \times ’, and the maximal basic types for any sets involved. Assume that S is a basic type below. Include a chain of reasoning using the \subset and $=$ relations. For example:

$$A \twoheadrightarrow B \subset A \twoheadrightarrow B \subset A \leftrightarrow B = \mathbb{P}(A \times B)$$

- (a) \mathbb{N}
 - (b) $\text{seq } S$
 - (c) $(\mathbb{N} \times \mathbb{N}) \rightarrow \mathbb{N}$
 - (d) $(\text{seq } S \times \text{seq } S) \rightarrow \text{seq } S$
2. Consider the following basic types for file names and bytes:

$[FILENAME, BYTE]$

Files may have read, write and execute permission:

$$PERM ::= r \mid w \mid x$$

The following schema models a directory of files:

<p><i>Directory</i></p> <p>$size : FILENAME \twoheadrightarrow \mathbb{N}$</p> <p>$permission : FILENAME \twoheadrightarrow (\mathbb{F} PERM)$</p> <p>$contents : FILENAME \twoheadrightarrow (\text{seq } BYTE)$</p> <p>$files : \mathbb{F} FILENAME$</p> <hr/> <p>$files = \text{dom } size = \text{dom } permission = \text{dom } contents$</p> <p>$(\forall f : files \bullet size f = \#(contents f))$</p>

- (a) Explain the above schema, line by line, using technical terms.
- (b) Explain the meaning of the schema in natural English suitable for someone who does not understand Z.
- (c) Normalize the schema by defining all the types using ‘ \mathbb{P} ’, ‘ \times ’, and maximal basic types.
- (d) Write an initialization schema called *InitDirectory* which starts the directory with no files in it.
- (e) Specify an operation schema *CreateFile* which adds a new file called $f?$ to the directory. The file should be empty and have read/write permission.
- (f) Specify another operation schema *DeleteFile* which deletes an existing file called $f?$ from the directory.